

THE DIGITAL TWIN · VOLUME 1  
BOOK 1 · THEORY

# The Digital Twin Theory & Foundations

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*Principles, Mathematical Models, AI, Standards, and the Science  
Behind Every Digital Twin*

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# Table of Contents

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Preface.....	6
How to Read This Suite.....	7
Part I — The Idea and Its Foundations.....	1
Chapter 1 — What Is a Digital Twin?.....	2
1.1 A Definition Worth Getting Right.....	2
1.2 Model, Shadow, Twin.....	2
1.3 A Short History.....	3
1.4 Why the Idea Unifies Engineering.....	4
Review Questions — Chapter 1.....	5
Chapter 2 — The Anatomy of a Digital Twin.....	6
2.1 A Reference Architecture.....	6
2.2 The Seven Layers.....	7
2.3 Reading the Stack Both Ways.....	7
2.4 The Architecture as Software.....	8
Review Questions — Chapter 2.....	9
Chapter 3 — The Mathematics of Modelling.....	10
3.1 Models Are Differential Equations.....	10
3.2 Three Equivalent Forms.....	10
3.3 Reading a First-Order Response.....	11
3.4 Discretisation: From Continuous to Computable.....	12
3.5 From Equation to Code.....	12
Review Questions — Chapter 3.....	14
Chapter 4 — Simulation Foundations.....	15
4.1 Why We Simulate.....	15
4.2 Numerical Integration.....	15
4.3 Step Size and the Central Trade-Off.....	16
4.4 Real-Time Simulation.....	17
4.5 The Two Solvers, Side by Side.....	17
Review Questions — Chapter 4.....	19
Part II — Modelling Physical Systems.....	20
Chapter 5 — Modelling Electromechanical Systems.....	21
5.1 Two Physical Domains, One Machine.....	21
5.2 The Electrical Equation.....	21
5.3 The Mechanical Equation.....	22
5.4 Coupling into a Second-Order Model.....	22
5.5 Reduction to First Order.....	23
5.6 From Model to Simulation.....	23
Review Questions — Chapter 5.....	25
Chapter 6 — Modelling Thermal, Optical, and Sensor Systems.....	26
6.1 The Same Equation, Different Clothes.....	26
6.2 Thermal Systems.....	26
6.3 Optical and Sensing Systems.....	27
6.4 Static Characteristic versus Dynamic Response.....	27
6.5 Linearisation.....	27
6.6 Calibration as Computation.....	28
Review Questions — Chapter 6.....	29

Chapter 7 — System Identification Theory.....	31
7.1 The Parameters Are Not Given.....	31
7.2 Identification as Least Squares.....	31
7.3 Maximum Likelihood.....	32
7.4 Informative Experiments.....	33
7.5 The Least-Squares Algorithm.....	33
Review Questions — Chapter 7.....	34
Chapter 8 — Control Theory for Twins.....	36
8.1 From Predicting to Acting.....	36
8.2 The PID Controller.....	36
8.3 Stability.....	37
8.4 Root Locus and Frequency Response.....	38
8.5 The PID Algorithm in Discrete Time.....	38
Review Questions — Chapter 8.....	39
Part III — Intelligence and Data.....	41
Chapter 9 — State Estimation and Sensor Fusion.....	42
9.1 Why Estimation Is Necessary.....	42
9.2 The Predict-Correct Cycle.....	42
9.3 Weighing Trust.....	43
9.4 Sensor Fusion.....	43
9.5 When the World Is Not Linear.....	44
Review Questions — Chapter 9.....	46
Chapter 10 — AI and Machine Learning in Digital Twins.....	47
10.1 Three Ways to Build a Model.....	47
10.2 Surrogate Models.....	48
10.3 Learning to Detect Anomalies.....	48
10.4 Physics-Informed Neural Networks.....	49
10.5 An Unsupervised Detector in Practice.....	49
Review Questions — Chapter 10.....	50
Chapter 11 — Data, Telemetry, and the IoT Layer.....	52
11.1 The Layer That Carries Everything.....	52
11.2 Edge, Fog, and Cloud.....	52
11.3 Two Patterns of Communication.....	53
11.4 The Three Constraints That Shape Every Link.....	53
11.5 A Receiver That Expects to Fail.....	54
Review Questions — Chapter 11.....	55
Part IV — Industrial Reality.....	57
Chapter 12 — Standards and Interoperability.....	58
12.1 Why Standards Matter.....	58
12.2 Four Standards That Matter.....	58
12.3 ISO 23247: a Framework for the Whole.....	59
12.4 The Asset Administration Shell.....	59
12.5 OPC-UA and IEC 61131-3.....	60
Review Questions — Chapter 12.....	61
Chapter 13 — Cybersecurity and Trust.....	62
13.1 The Risk Is Physical.....	62
13.2 IT and OT Have Opposite Priorities.....	62
13.3 Defence in Depth.....	63
13.4 IEC 62443.....	63
Review Questions — Chapter 13.....	65

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Chapter 14 — Industrial Applications and Case Studies.....	66
14.1 One Method, Many Industries.....	66
14.2 A Tour of the Domains.....	66
14.3 The Common Pattern of Value.....	67
14.4 Scale Changes Everything but the Method.....	67
Review Questions — Chapter 14.....	68
Part V — Frontiers.....	70
Chapter 15 — Open Problems and Research Directions.....	71
15.1 The State of the Art.....	71
15.2 Cognitive Twins.....	72
15.3 Fleet and Federated Twins.....	72
15.4 The Standardisation Gap.....	72
15.5 Trust, Ethics, and Explainability.....	72
15.6 Sustainability.....	73
15.7 The Road Ahead.....	73
Review Questions — Chapter 15.....	74

## Part I — The Idea and Its Foundations

*Part I establishes what a digital twin is, the anatomy every twin shares, the mathematics of modelling a physical system, and the numerical foundations that let a model run.*

## PART I — THE IDEA AND ITS FOUNDATIONS

## Chapter 1 — What Is a Digital Twin?

### Learning Objectives

**By the end of this chapter you will be able to:**

Define a digital twin precisely, and distinguish it from a digital model and a digital shadow.

Trace the origin of the idea from the Apollo programme to modern Industry 4.0.

Identify the three properties an information system must have to qualify as a true twin.

Explain why the digital twin is a unifying idea across many engineering disciplines.

### 1.1 A Definition Worth Getting Right

A digital twin is a virtual representation of a physical asset that is kept in correspondence with that asset, throughout its life, by an automatic flow of data between the two. Every word in that sentence earns its place. It is a representation, not the asset itself. It is virtual, living in software. It is kept in correspondence — not built once and abandoned, but continually reconciled with reality. That reconciliation is automatic, not a person occasionally updating a spreadsheet. And it persists throughout the life of the asset, ageing as the asset ages.

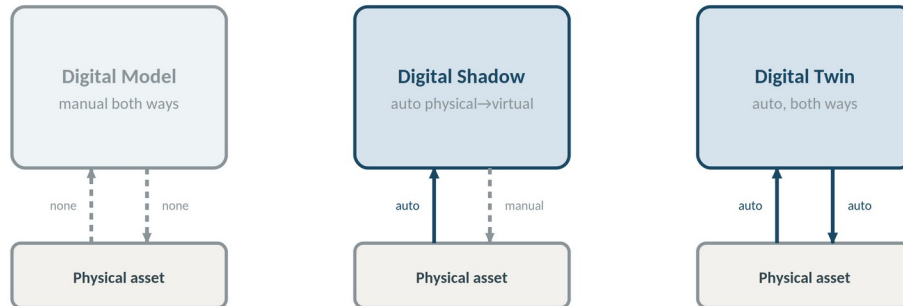
This definition is stricter than common usage, and deliberately so. Much of what industry calls a digital twin is, on close inspection, something less. Drawing the line precisely is the first task of a theory of digital twins, because a concept that means everything means nothing.

### 1.2 Model, Shadow, Twin

Michael Grieves, who named the concept, drew a distinction that has proved durable. It rests entirely on how data flows between the physical and the virtual.

## Digital model, digital shadow, digital twin

What distinguishes a true twin is automatic, two-way data flow.



**Figure 1.1. Digital model, digital shadow, digital twin.** *What separates them is whether the data link is manual or automatic, and one-way or two-way.*

A **digital model** is a virtual representation with no automatic data link at all. A designer's CAD model or a hand-tuned simulation is a digital model: changing the asset does not change the model, and changing the model does not change the asset. Any updating is manual.

A **digital shadow** adds a one-way automatic link. Data flows from the physical asset to the virtual representation automatically, so the shadow tracks the asset in something like real time — but the asset receives nothing back. A live dashboard fed by sensors is a digital shadow.

A **digital twin** closes the loop. Data flows automatically in both directions: the asset updates the twin, and the twin can act back on the asset — adjusting a setpoint, raising an alarm, scheduling maintenance. This two-way, automatic, lifelong link is the defining property, and it is what makes a twin an instrument of action rather than merely of observation.

### Box 1.1 — The test, in one question

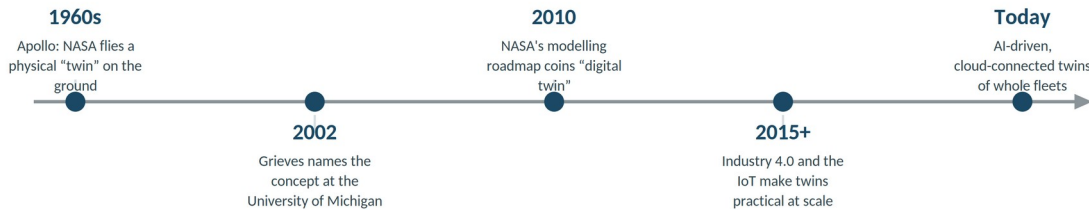
*To decide what you are looking at, ask: can the virtual representation change the physical asset, automatically, without a human in the loop? If no data flows at all, it is a model. If data flows only from asset to representation, it is a shadow. If data flows both ways, automatically, it is a twin. Most “digital twins” marketed today are, by this test, shadows — which is no criticism, only a clarification.*

### 1.3 A Short History

The idea is older than the name. During the Apollo programme, NASA maintained physical duplicates of spacecraft on the ground, updated continuously with

telemetry from orbit, so that engineers could rehearse decisions on an Earth-bound copy of the vehicle in space. When Apollo 13 was crippled, it was these ground twins that let engineers test fixes before radioing them to the crew. The medium was physical, but the logic was already that of a twin: a maintained, parallel copy used to reason about the original.

### A short history of the digital twin idea



**Figure 1.2.** A short history of the digital twin idea. *From physical duplicates in the 1960s to AI-driven fleet twins today.*

Michael Grieves introduced the conceptual model in 2002, in the context of product lifecycle management, and NASA's 2010 modelling roadmap gave the term "digital twin" its modern currency. What turned a concept into a practice, however, was the convergence of cheap sensors, ubiquitous connectivity, and abundant computing that arrived with the Internet of Things and Industry 4.0. Only then did it become economical to instrument an asset densely enough, and connect it reliably enough, for a true two-way twin to pay for itself.

#### 1.4 Why the Idea Unifies Engineering

The digital twin is one of those rare ideas that draws disciplines together rather than dividing them. Building one demands a model of the physical asset, which is the province of the mechanical, electrical, or chemical engineer. It demands sensing and signal processing, the province of the instrumentation engineer. It demands communication networks, the province of the network engineer. It demands modelling, control, and increasingly machine learning, the province of the systems and software engineer. No single discipline can build a twin alone, and that is precisely why the subject is worth a unified theory.

→ **Build it** Book 2, Chapters 6–11. *The abstract twin defined here becomes concrete in Book 2, where a DC motor is twinned end to end — measured, modelled, and acted upon in a closed two-way loop.*

## Chapter Summary

- A digital twin is a virtual representation kept in automatic, two-way correspondence with a physical asset throughout its life.
- A digital model has no automatic link; a digital shadow has a one-way link; only a twin closes the loop both ways.
- The idea dates to NASA's Apollo-era physical duplicates; the name and modern practice arrived with Grieves, NASA, and Industry 4.0.
- Building a twin requires modelling, sensing, communication, control, and data science together, which is why the field is inherently multidisciplinary.

## Key Terms

Term	Meaning
Digital model	A virtual representation with no automatic data link to the asset.
Digital shadow	A representation with a one-way automatic link from asset to model.
Digital twin	A representation with an automatic, two-way link to the asset, maintained for its life.
Industry 4.0	The wave of industrial digitisation that made large-scale twinning practical.

## Review Questions — Chapter 1

### Recall

- 1.1 State the defining property that separates a digital twin from a digital shadow.
- 1.2 What role did ground-based duplicates play in the Apollo programme?

### Application

- 1.3 For a live sensor dashboard of a wind turbine, classify it as model, shadow, or twin, and say what would have to be added to promote it to the next category.

### Analysis

- 1.4 Explain why dense sensing and reliable connectivity were preconditions for true twins, using the model/shadow/twin distinction.

### Design

- 1.5 Choose a familiar machine and sketch, in words, the data flows that would make a genuine two-way twin of it, identifying the action the twin could take on the asset.